The Pursuit of Mathematical Truths: A Rich and Meaningful Aesthetic Experience of Inquiry as an End in Itself

Kevin Ulliac

In this paper, I will argue that the pursuit of mathematical truths can be classified as rich and meaningful aesthetic experiences that are ends in themselves, and I will demonstrate this by using John Dewey's philosophy of mathematics, his philosophy of education, and what he takes to be the nature of experience in order to substantiate my position. My argument is that mathematical experiences of inquiry can have a meaningful impact on all individuals; this experience is not unlike one's admiration for works of art, such as a painting, or a novel, or a piece of music. The nature of mathematical inquiry, especially in the context of scientific endeavors, is to expand an individual's understanding of the nature of reality and to expand their conscious experience of themselves, their relation to other people, and their relation to the world at large. Furthermore, all modes of inquiry and the experiences that result are educative experiences that are not merely a means to some end, but they are ends in themselves. The expansion and perpetuation of human flourishing via experiences of inquiry provides an aesthetic experience that is good for its own sake. All pursuits of complete intellectual experiences are moments of learning which expand meaning and conscious experience and are therefore ends in themselves.

In *Experience and Nature* and *Art as Experience*, Dewey explicitly argues that there are two kinds of possible worlds devoid of meaning, namely, a world in constant flux and a world that is never changing. In neither of these worlds is there possibility for meaning or aesthetic experiences. Dewey explains that in a world of constant flux, everything would be in chaos; everything that came into being and everything that happens would be purely arbitrary—there would be no such thing as cause and effect. In a world that is never changing, everything that existed would have always existed and will continue to exist for eternity; once again there would be no such thing as cause and effect. But a world that is in equilibrium of the two affords us a world of both stability *and* change, out of which affords the possibility for cause and effect, which then provides the possibility of meaning—and only in this composite world could we develop mathematical and scientific means to obtain desired ends.[1]

Here is a quote from Dewey's *Experience and Nature* that will help to illustrate his philosophy of mathematics:

> They are out of time in the sense that a particular temporal quality is irrelevant to them. If anybody feels relieved by calling them eternal, let them be called eternal. But let not 'eternal' be then conceived as a kind of absolute perduring existence or Being. It denotes just what it denotes: irrelevance to existence in its temporal quality. These non-temporal,

[1] John Dewey, The Later Works of John Dewey, 1925-1953 (Vol 1): 1925, *Experience and Nature*, ed. J. Boydston, (Carbondale, IL: Southern Illinois University Press, 2008), 59-61.
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John Dewey, The Later Works of John Dewey, 1925–1953 (Vol. 10): 1934, *Art as Experience*, ed. J. Boydston, (Carbondale, IL: Southern Illinois University Press, 2008), 22-3.

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> mathematical or logical qualities are capable of abstraction, and of conversion into relations, into temporal, numerical and spatial order. As such they are dialectical, non-existential.[2]

Dewey's philosophy of mathematics describes the totality of mathematics as a set of non-existential, atemporal qualities that are capable of abstraction and formulation. He also recognizes that mathematics has instrumental value. He writes, "But also as such they are tools, instrumentalities applicable to historic events to help regulate their course."[3] Few would argue with Dewey that mathematics can have instrumental applications and is thus instrumentally valuable. However, mathematics is more than a tool, it is a formal and symbolic language—a medium under which we derive meaning, and as such, it is a valuable mode of inquiry that provides meaningful solutions to meaningful problems. Moreover, I hold that mathematics is like natural languages insofar as it is a tool of tools, and that the means of mathematical inquiry is not separate from the ends—there is continuity between the doing of mathematics and the valuable ends it produces. Just as language provides us a medium for inquiry and thought, so too does mathematics; consequently, mathematical inquiry houses meaningful experiences just as much as speaking.

I'll further elaborate on the nature of mathematics to better understand its limits and its powers. Mathematics and logic, being formal and symbolic languages, have a set of symbols that are arranged into 'sentences' that are governed by a given set of syntactical rules. For example, the symbol '1' refers to the abstract concept of one complete unit. It doesn't refer to any concrete object of experience necessarily, though it certainly can be applied as such. When I refer to, say, 'an apple,' I'm implicitly referring to a single and whole object that is subsumed under the category 'apple,' which may or may not refer to an actual apple in the real world (which exists in space and in time); I could also simply be referring to the concept 'apple,' which is non-existential in its own right—in this sense it has no physical referent. In *Logic: The Theory of Inquiry*, Dewey explains:

> The interpretation of the logical conditions of mathematical conceptions and relations must be such as to account for the form of discourse which is intrinsically free from the necessity of existential reference while at the same time it provides the possibility of indefinitely extensive existential reference-such as is exemplified in mathematical physics.[4]

Mathematics *must* hold abstract operational power; it must have the ability to refer to something existential in the world *and* the ability to refer to nothing in particular—i.e., without physical referent. To further elaborate the previous example, the '+' symbol is used to combine units, thus representing a given abstract concept of assembling two or more groups of things into one newly established group of things, arbitrarily defined. When I have, say, 'three apples', I can represent this group of apples with the number '3', which refers to three existential entities (the apples). I can also represent it arithmetically as $1 + 1 + 1 = 3$. The numbers and arithmetic operations are symbols that represent a given concept, and put together in the example above, we have a coherent arithmetic 'sentence' that also happens to be true. If I were to represent this

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[2] Dewey, *Experience and Nature*, 119.
[3] Ibid., 119.
[4] John Dewey, *Logic: The Theory of Inquiry*, (New York, NY: Henry Holt and Company, 1938), 394.

as 111 = 3, then I've clearly not understood mathematical syntax, nor have I understood how to formulate a coherent representation of the concept of adding three individual units together and to represent them as totaling to three. Mathematics and logic, just like any natural language, have a set of symbols and operations, which are combined to form 'sentences' that are governed by a given set of syntactical rules. The 'sentences' necessarily convey meaning, for they always have a referent (physical or non-physical).

Having sufficiently established the nature of mathematics, I find it crucial to illustrate how Dewey describes the nature of human experience. In *Art as Experience*, Dewey offers the following description:

> The nature of experience is determined by the essential conditions of life. While man is other than bird and beast, he shares basic vital functions with them and has to make the same basal adjustments if he is to continue the process of living. Having the same vital needs, man derives the means by which he breathes, moves, looks and listens, the very brain with which he coordinates his senses and his movements, from his animal forbears. The organs with which he maintains himself in being are not of himself alone, but by the grace of struggles and achievements of a long line of animal ancestry.[5]

Human experience is governed by the same essential conditions for life as any other animal. We are governed by the same laws of nature and we require the same natural resources (food, shelter, water, etc.) to survive. We even have very similar bodily structures such as brains, organs, limbs, digestive systems, and the like. Inventing and systematizing agricultural methods helps us secure food; constructing insulated housing helps us maintain living arrangements and protects us from the seasonal elements; settling next to a source of fresh water is a sure way of guaranteeing the survival of one's society. Let me illustrate the following: For an agricultural civilization to succeed, they need to understand how vegetation grows and flourishes. They also need to understand how the solar cycles and seasons work because one must plant seeds at a certain time of the year to allow enough warm weather and sunshine for their plants to grow and be successfully harvested— essentially, the civilization must recognize that solar cycles are consistent, they must sufficiently understand how these solar cycles work, and only then are they able to predict the seasons.

This is where mathematics comes in. In *A Short History of Astronomy*, Arthur Berry provides a very good description of the seasons from the perspective of an inquisitive observer who's trying to understand the sun's patterns throughout the solar year. He writes:

> The sun is daily seen to rise in the eastern part of the sky, to travel across the sky, to reach its highest position in the south in the middle of the day, then to sink, and finally to set in the western part of the sky. But its daily path across the sky is not always the same: the points of the horizon at which it rises and sets, its height in the sky at midday, and the time from sunrise to sunset, all go through a series of changes, which are accompanied by changes in the weather, in vegetation, etc.; and we are thus able to recognize the existence of the seasons, and their recurrence after a certain interval of time which is known as a year.[6]

He provides a similar description regarding the phases of the moon:

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[5] Dewey, *Art as Experience*, 19.

[6] Arthur Berry, *A Short History of Astronomy*, (New York, NY: Dover Publications, 1961), 2-3.

> The different forms thus assumed by the moon are now known as her phases; the time occupied by this series of changes, the month, would naturally suggest itself as a convenient measure of time; and the day, month, and year would thus form the basis of a rough system of time-measurement.[7]

An observer must first recognize that the sun and moon have cycles, and then they must devise ingenious methods of measurement that can accurately keep track of the solar and lunar cycles. For tens of thousands of years, humans have been keeping track of solar and lunar cycles with the use of mathematics and geometry (and with clever methods of engineering too). Archaeologists have found thousands of engraved mammoth tusks that date roughly 30,000 years old, and are said to be the records of lunar cycles in the Paleolithic era.[8] There are also several sophisticated and elaborate ancient structures that keep track of the seasons by measuring the amount of daylight, which in turn allows one to accurately describe and predict the summer and winter solstice, fall and spring equinox, as well as keeping track of lunar cycles.[9]

The engraved mammoth tusks, Stonehenge, and the Great Kiva are all primitive calendars—they are time keeping machines. In *Mapping Time*, E.G. Richards explains very succinctly what a calendar does. He writes:

> All calendars are based on the succession of days and nights punctuated by the waxing and waning of the moon or the rhythm of the seasons and the movement of the sun. In the beginning, our remote ancestors had no traditional knowledge of the regularity of the motions of the sun and the moon to guide them. Only after they had learnt to count and do simple arithmetic, and after many nights of careful observation of the heavens, did the calendar begin to take shape. Before that, they had no calendar to help them plan ahead or organize their experience of time—just a succession of days and months and seasons.[10]

Calendars are incredibly powerful inventions since they allow us to accurately keep track of the days, months, and seasons in a given year. What's important to understand is that to create a functional calendar system, a civilization must first devise a theory of number, must be able to count and to perform basic arithmetic operations, and must also be able to make accurate observations by using accurate and intelligent methods of measurement— the society must first devise a sophisticated system of mathematics and geometry, as well as a sophisticated means of engineering the tools of measurement, and then they must accurately collect data, perform calculations with minimal error, and they must interpret the resulting data appropriately.

I will now turn your attention to what Dewey describes as an "aesthetic experience." In *Art as Experience*, Dewey proposes the following: "Only when the past ceases to trouble and anticipations of the future are not perturbing is a being wholly united with this environment and therefore fully alive."[11] In relation to the development of the calendar, civilizations of the past

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[7] Berry, *A Short History of Astronomy*, 3.

[8] James E. McClellan III and Harold Dorn, *Science and Technology in World History*, (Baltimore, MD: The Johns Hopkins University Press, 2006), 15.

[9] Many ancient structures remain intact to this day; structures such as the infamous Stonehenge in England and the Great Kiva at Pueblo Bonito in New Mexico.

McClellan and Dorn, *Science and Technology in World History*, 27.

[10] E.G. Richards, *Mapping Time: The Calendar and its History*, (New York, NY: Oxford University Press, 1998), 3-4.

[11] Dewey, *Art As Experience*, 24.

recognized that the ability to fully understand and calculate the solar, lunar, and seasonal cycles brings tremendous value for securing essential goods and for anticipating seasonal changes—especially notable is the importance of an accurate and comprehensive working knowledge of astronomy, which can be directly applied to agricultural practice, which in turn helps to ensure a successful harvest of one's crops. With the mathematical understanding of how the solar, lunar, and seasonal cycles function, a civilization can secure their required supply of food for the year. With mathematical inquiry, civilizations of the past could devise calendrical systems, thus, their troubles of the past were diminished and their future secured. The advent of the calendar—which is achieved solely through the understanding of mathematics and its application in astronomy— is largely responsible for civilizations of the past being *united* with their environment as opposed to being subservient to it. The civilizations of the past were free to enjoy the potential riches that their environment affords. Thus, mathematical experiences of inquiry into the nature of the solar, lunar, and seasonal cycles certainly *allow* for the possibility of having an aesthetic experience. My argument goes one step further, namely, I argue that mathematical experiences of inquiry are themselves aesthetic experiences.

One need not, however, understand the entire workings of the solar system to have an aesthetic experience. Dewey also describes how all experiences, regardless of their form, can lead to an aesthetic experience. He writes: "Because experience is the fulfillment of an organism in its struggles and achievements in a world of things, it is art in germ. Even in its rudimentary forms, it contains the promise of that delightful perception which is esthetic experience."[12] Many of the struggles that an organism experiences can be alleviated by various means and employing the mathematical method of inquiry is one way to secure future outcomes. I have sufficiently demonstrated this with the example of the development of the calendar—the calendar helps to secure resources for the survival of the organism. The calendar is a tool, but the mathematical method of inquiry that led to the invention of the calendar is itself a tool of tools. The mathematical method of inquiry helps to cultivate meaning and contributes to the expansion of human flourishing and the expansion of conscious experience of the world we inhabit. The calendar is not merely a means to the successful planting and harvesting of crops but is itself a part of the process of developing an agrarian lifestyle. Mathematics is fully embedded in all activities that provide meaning and purpose to our existence and to the perpetuation and flourishment of our species. It is both a product of our lived projects and it provides the necessary structure for those projects.

In the same way that an individual can have an aesthetic experience while admiring a work of art or by reading a piece of poetry, an individual can have an aesthetic experience while investigating a mathematical formula that expresses something meaningful about the world; Dewey would agree with this statement. In *Art as Experience*, he writes:

> I have referred more than once to the esthetic quality that may inhere in scientific work. To the layman, the material of the scientist is usually forbidding. To the inquirer there exists a fulfilling and consummatory quality, for conclusions sum up and perfect the conditions that lead up to them. Moreover, they have at times an elegant and even austere form.[13]

[12] Ibid., 25.
[13] Ibid., 202.

Take for instance Newton's equation F = ma and Einstein's equation E = mc2. Both equations have deep philosophical meaning, both in what the symbols represent individually and in their relation to each other. Newton believed that acceleration was only possible if space was an absolute substance—his equation of F = ma reflects the entirety of his ontological view of space and of accelerated motion in such a space. Einstein held that all matter and energy are interchangeable and thus can be converted from one to the other. His equation reflects his ontological view of matter and energy; namely, that they are essentially one and the same.

The logical notion of Modus Tollens and the Copernican model of the universe are also examples of mathematical symbols having a great deal of meaning within the equations, the symbols therein, and within the philosophical conclusions they express. Modus Tollens is essentially a logical rendering of the principle of cause and effect. It states that: "If **P** then **Q**. Since ~**Q**, therefore ~**P**." In other words, if it's true that **P** always causes **Q**, and if it is the case that **Q** has not obtained, then it is also the case that **P** has not obtained.[14]

The Copernican Heliocentric model of the universe and of our solar system is represented and substantiated mathematically, but it has deep philosophical principles embedded within. Ptolemy, a Greek astronomer who lived in the middle of the 2nd century C.E., worked out a geocentric mathematical model of the universe. In so doing, he was trying to mathematically quantify exactly how the sun, the moon, and the planets move in the night sky in relation to the earth. In his model, he corrected for the apparent retrograde motion of the planets by formulating various epicycles—it was well known that all the visible stars and planets appear to move in the night sky from east to west throughout the evening, but then for a short period (weeks or months) the planets would slow down, retract in the opposite direction (west to east), slow down once again and return to their original state of motion (east to west). Ptolemy made several mathematical calculations and determined that the planets were not only in circular motion around the earth but had an additional state of circular motion within its orbit around the earth—an orbit within an orbit.[15]

Copernicus published *De Revolutionibus*[16] in 1543 in response to his deep dissatisfaction with Ptolemy's model. He thought Ptolemy's model "failed to provide a satisfactory account of the stations and retrogradations of the planets; with its elaborate geometrical constructions, it was an astronomical 'monster'."[17] Copernicus sought to significantly reduce the number of epicycles by putting the sun at the centre of our solar system. He did nothing more than to simplify Ptolemy's model by hypothesizing a heliocentric model—the earth was no longer the center of the universe.[18] Copernicus assumed a heliocentric model of the universe without empirical evidence. As McClellan and Dorn write: "He simply hypothesized heliocentrism and worked out his astronomy from there […] [Copernicus] made these bold assumptions for essentially aesthetic and ideological reasons."[19] Though his model had little empirical evidence, Copernicus' model was far simpler and had a natural explanation for the retrogradations of the planets. The stations and retrogradations are simply an illusion

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[14] Merrie Bergmann, *The Logic Book*, (New York, NY: McGraw Hill, 2014), 214-6.
[15] Arthur Berry, *A Short History of Astronomy*, (New York, NY: Dover Publications, 1961), 63-72.
[16] *On the Revolutions of the Heavenly Spheres* is the English translation of the title.
[17] McClellan and Dorn, *Science and Technology in World History*, 208-9.
[18] Berry, *A Short History of Astronomy*, 99-107.
[19] McClellan and Dorn, *Science and Technology in World History*, 209.

resulting from the relative motion of the earth and the planets against the background of the fixed stars. McClellan and Dorn explain: “With heliocentrism the appearance of the stations and retrogradations of the planets remains, but the problem vanishes: ‘retrograde’ motion automatically follows from the postulate of heliocentrism.”[20]

Ptolemy’s geocentric model of the universe had a deeply implicit assumption, namely, that the earth is stationary and everything we observe moving across the sky is circling around us, including the sun, moon, and stars. The implicit assumption was, by extension, that the earth therefore must be the center of the universe. Ptolemy’s calculations were directly derived from this implicit assumption. Copernicus flipped that assumption on its head, grounding his own assumptions not on the raw perceptions and observations of what we observe in the sky, but rather on something more fundamental, something that has great philosophical importance; namely, mathematical simplicity and completeness. Copernicus hypothesized that “the earth rotates once a day on its axis, thus accounting for the apparent daily motion of everything in the heavens, and the earth revolves around the sun once a year, accounting for the sun’s apparent annual motion through the heavens.”[21] With a simple shift in perspective and a generous employment of mathematical calculations, Copernicus took Ptolemy’s convoluted geocentric model and turned it into a much simpler and more aesthetic *heliocentric* model, all while using the same observational data that Ptolemy used.

An attempt to understand how the universe operates is a deeply philosophical endeavor, one that requires more than just rhetoric and observation. One must employ mathematics, geometry, and the totality of their understanding of physics and astronomy. Thus, to understand the universe and the motions of the heavenly bodies, one must employ a rigorous set of mathematical and geometrical modes of inquiry that yield a mathematical and geometric answer. No other mode of inquiry yields a satisfactory answer. This again demonstrates the power of mathematics and its special ability to enhance meaning and conscious experience. When an inquisitive observer employs their understanding of mathematics, geometry, physics, and astronomy wisely, they can answer meaningful questions about the nature of reality and of the world they inhabit.

I will now borrow from Pollard’s essay, “Mathematics and the Good Life.” He writes: “Creative mathematical experiences are not only intrinsic goods: they are goods that live again in re-creations that are themselves intrinsically good.”[22] Creative mathematical experiences are re-creations in the sense that, when an individual teaches another how to solve a mathematical problem, the teacher is, in effect, evoking in the student the same experience of inquiry that the teacher originally had while solving said problem. Pollard further describes the contributions that mathematics makes to human prosperity: “Mathematics pervades scientific explanations and, as we have seen, is also an engine of understanding, a propagator of insightful experiences, in its own right. Mathematics has additional ethical significance because it makes further contributions to human prosperity.”[23] Anything that contributes to human prosperity is an

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[20] Ibid., 210.
[21] Ibid., 210.
[22] Stephen Pollard, “Mathematics and the Good Life,” *Philosophia Mathematica* (III) 21, no. 1 (December 2013): 102, https://doi.org/10.1093/philmat/nks030.
[23] Pollard, “Mathematics and the Good Life,” 106-7.

intrinsic good. Mathematics, given its foundational properties and its stature in scientific explanations and insights in general, is such an intrinsic good. Pollard writes:

> To say mathematicians are in the problem-solving business is, from Dewey's distinctive perspective, to say they are in the business of educative experience — and that makes mathematics an affair of the deepest ethical significance. A coherent progression of ever richer experiences drawing on an expanding store of developing capacities: that is the good for a human being.[24]

To further illustrate the importance of mathematical inquiry and its ethical significance, I turn to Dewey's book *Democracy and Education*, where he defines education as an end in itself. Education is that which brings about a "widening and deepening of conscious life, a more intense, disciplined, and expanding realization of meanings."[25] For Dewey, education is the expansion of meaning and conscious experience, thus education in all forms is an end in itself. My argument is that a mathematical experience of inquiry is itself a highly educative process, whereby the inquirer reflects upon the nature of reality; the means of mathematical inquiry is part of the educative process at large. Pollard argues similarly; he writes: "The socially valuable something for which a mathematician is good is, precisely, the dissemination of tools for the evolution of conscious life. Mathematicians have educative experiences: experiences that expand the capacity for educative experience."[26] A mathematical experience of inquiry is a moment of learning, but it can also be an aesthetic experience, especially when such an experience expands meaning and consciousness in general.

Furthermore, the means of all mathematical inquiry is not separate from the ends. In *How We Think*, Dewey presents a compelling definition of thought. He writes: "Thoughts that result in belief have an importance attached to them which leads to reflective thought, to conscious inquiry into the nature, conditions, and bearings of the belief."[27] Thoughts that have any importance to the organism and to the relation to their environment lead to reflexive thought and beliefs about the nature of the universe. A thought of this kind includes, but is not limited to, a working model of the universe (i.e., our current heliocentric model of the solar system) and the understanding of scientific and logical principles such as F = ma, E = mc2, and Modus Tollens; all of these are products of reflexive thought on the nature of the universe—as such, these products of reflexive thought are densely rich in meaning and most individuals will surely have an aesthetic experience when investigating these principles. In *Art as Experience*, Dewey describes the link between intellectual inquiry and an aesthetic experience. He writes:

> What is even more important is that not only is this quality a significant motive in undertaking intellectual inquiry and in keeping it honest, but that no intellectual activity is an integral event (is an experience), unless it is rounded out with this quality. Without it, thinking is inconclusive. In short, esthetic cannot be sharply marked off from intellectual experience since the latter must bear an esthetic stamp to be complete.[28]

As Dewey explains, an intellectual experience—whether it be a mathematical experience of inquiry, or any other experience of inquiry—must have an aesthetic quality to it for this

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[24] Ibid., 103.
[25] John Dewey, *Democracy and Education*, (New York, NY: The Macmillan Company, 1930), 417.
[26] Pollard, "Mathematics and the Good Life," 103.
[27] John Dewey, *How We Think*, (New York, NY: D.C. Heath & Co., 1910), 5.
[28] Dewey, *Art as Experience*, 45.

experience to be complete. As I have hopefully demonstrated, mathematical experiences of inquiry are complete aesthetic experiences that expand meaning and conscious experience; they perpetuate human flourishment, and as such, they are ends in themselves.

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