Predicting the Future of Human-Coyote Interactions in Edmonton Using Differential Equations

Sarah Marklund

Abstract

In this paper, a system of differential equations is formulated to study the issue of human-coyote interactions in Edmonton. The system's asymptotic behaviour is examined to predict the size of the coyote population in the future, as well as the number of bold coyotes and individuals concerned about coyotes. These values are shown to stabilize, indicating a need for better management to fully eliminate bold coyote behaviour.

Introduction

Though human expansion has had negative consequences for many species, coyotes have shown no struggle in being able to live in urban environments [13]. However, humans have not remained similarly unaffected by the presence of these animals, with conflicts between humans and coyotes only increasing as time goes on in various locations in North America [1], [7], [15]. The issue of coyote-human interactions has become serious enough to warrant multiple studies, each attempting to better understand these interactions in an effort to reduce conflicts [12], [15], [18].

The Edmonton Urban Coyote Project was one such study which focused on interactions between coyotes and people in Edmonton using data collected from people submitting online reports detailing their encounters with coyotes [7]. In this study, statistical methods were used to determine the degree to which various factors affected the behaviour of coyotes and reactions of people in human-coyote interactions. This included an analysis of changes in the level of coyote boldness and human concern about coyotes over a 10-year period [7]. This analysis involved using linear regressions with the data they gathered about the number of human-coyote interactions and the nature of each encounter over these 10 years [7]. The percentage of total reports from each year involving bold coyote behaviour and those involving human concern both increased significantly, indicating a rise in these two factors [7].

However, a differential equations approach has not been used to predict how these levels may look in the future. As such, we sought to create a system of differential equations to model the level of coyote boldness and human concern, as well as the size of the coyote population. Focusing on the future of this population and the impact of human-coyote interactions is important in informing how we choose to manage our urban coyote population.

We begin by detailing the system of differential equations we created. This system is partially based on the logistic model for the growth of a single population [6], as well as the Lotka-Volterra model for competition between two species [6]. These types of models are commonly used to better understand the behaviour of a single population considering the

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influence of different factors such as dispersal and migration [20], [17], as well as to examine the effects of different types of interactions between species [8], [19].

After describing our system, we then examine its behaviour. We focus on fixed points and asymptotic behaviour, using the eigenvalues of the Jacobian at each of these points to determine their stability. This behaviour is then interpreted in order to understand what the system predicts about the future of the coyote population and the levels of coyote boldness and human concern.

The paper is organized as follows. In Section 2, we present and fully explain our system, including the determination of the value of the parameters. Section 3 contains our examination of the fixed points of the system. Section 4 then focuses on the interpretation of these results. The paper's conclusion is given in Section 5.

Modelling Human-Coyote Interactions

The system of equations we will be working with is as follows:

$$\dot{C}(t) = a_1 \cdot C(t) - a_2 \cdot C^2(t) - a_3 \cdot BC(t) \cdot CH(t) \quad (1)$$

$$\dot{BC}(t) = b_1 \cdot BC(t) - b_2 \bullet BC^2(t) - b_3 \cdot BC(t) \cdot CH(t) + b_4 \cdot C(t) \quad (2)$$

$$\dot{CH}(t) = d_1 \cdot CH(t) - d_2 \bullet CH^2(t) - d_3 \cdot BC(t) \cdot CH(t) + d_4 \cdot C(t) \quad (3)$$

In this system, *C* represents the number of coyotes in Edmonton, *BC* represents the total number of bold coyotes, and *CH* represents the total number of individuals concerned about coyotes. We use the number of reports of bold coyotes and reports where an individual felt concerned as proxy values for *BC* and *CH* respectively. Due to a lack of data on the number of coyotes in Edmonton, we use the data about coyote population dynamics reported in [11].

The meaning of each coefficient and its value is presented in Table 1. Many parameters are very hard to determine due to difficulties in collecting extensive data about coyote populations, so in Table 1 we present a possible scenario based on historical data and data collected by the City of Edmonton.

Table 1. Coefficients of the Mathematical Model Given in Equations (1)-(3)

Coefficient	Meaning	Value
a_1	Per capita growth rate of the coyote population	0.6
a_2	Effect of intraspecific competition on growth rate	0.2
a_3	Effect of human-coyote interactions on growth rate	0.0025
b_1	Per capita growth rate of number of bold coyotes	0.33
b_2	Effect of reports of bold coyotes on growth rate	0.02
b_3	Effect of concerning human-coyote interactions on growth rate	0.01
b_4	Effect of coyote population size on the growth rate	0.05
d_1	Per capita growth rate of number of concerned individuals	0.85
d_2	Effect of reports where concern was expressed on growth rate	0.02
d_3	Effect of concerning human-coyote encounters on growth rate	0.01
d_4	Effect of coyote population size on the growth rate	0.08

The rationale behind the values of these coefficients is given below.

Equation 1

This equation models the growth of the coyote population. The first two terms of the equation are based on the classic logistic model for a single species population growth [6]. The value of a_1 is taken from literature on coyote population dynamics [11].

The term $BC \cdot CH$ accounts for lethal removal of certain problematic or particularly aggressive coyotes, which occurs in Edmonton in exceptional cases [4]. The coefficient of $BC \cdot CH$ is small as lethal removal of a small number of coyotes has a negligible effect on the population density [9], [5]. In addition, not all interactions between humans and coyotes create concern and contribute to coyote eliminations.

Equation 2

This equation was partially based on the equations for two competing species [6]. In this case, the two competing “species” are actually the numbers of bold coyotes and concerned individuals, with this equation describing the behaviour of the number of bold coyotes. We determined the coefficient of *BC* using data from the Edmonton Urban Coyote Project. Figure 6 from [7] shows 10 reports of bold coyotes in 2012 and a total of 682 by the end of 2021 [7], so we calculate the rate of growth as $\frac{672}{10} = 67.2$. We divide by the initial number of coyote reports to obtain a per capita growth rate of 0.33.

The coefficient of BC^2 is negative since as coyotes become bolder and create concern among people in Edmonton, there are increased efforts by the municipal government to educate the public on how to “haze” coyotes to reduce boldness [4], in addition to implementation of lethal removal [4]. The value of this coefficient is small due to possible inconsistencies in the implementation [2] and efficacy [2], [3] of hazing. The coefficient of $BC{\cdot}CH$ is small for similar reasons.

The term *C* accounts for how the size of the coyote population affects the number of bold coyotes. We use data from the Edmonton Urban Coyote Project [7] to estimate that 5% of the coyote population is bold as some of their reports of bold coyotes may be describing the same coyote.

Equation 3

Equation 3 is once again partially based on the Lotka-Volterra competition model [6]. This equation describes the behaviour of the second “species”, which in this model is represented by the number of reports of human-coyote interactions where an individual felt concerned. The coefficient of *CH* was determined using Figure 6 from [7] in the same way as the coefficient of *BC* in Equation 2.

The term CH^2 accounts for the effect of individuals who try to educate others and reduce people’s concern in an effort to protect coyotes. For simplicity, the value of the coefficient of CH^2 was made the same as that of the coefficient of BC^2. The value of the coefficient of $BC{\cdot}CH$ was also kept the same as in Equation 2.

The coefficient of *C* was determined in the same way as in Equation 2.

By using the values given above, we hope to obtain results similar to those found in [7].

System Behaviour

We use Maple to examine the behaviour of our system according to the values of the coefficients given in Section 2. We first identify its fixed points.

Fixed points represent equilibrium points for the system. If a fixed point is stable and initial conditions are in the neighbourhood of the point, then the system will go toward this stable fixed point as time goes on. If a fixed point is unstable, then the system will go away from it no matter how close the initial conditions are to this unstable fixed point [16]. Thus, the fixed points of our system are of interest as they represent possible future scenarios regarding the size of the coyote population and levels of boldness and concern.

Let the functions f_1, f_2, f_3 be given by the expressions on the right-hand sides of equations (1), (2), (3), respectively. The fixed points are the roots of the system

$$f_1(C, BC, CH) = 0.6 \cdot C(t) - 0.2 \cdot C^2(t) - 0.0025 \cdot BC(t) \cdot CH(t) = 0$$

$$f_2(C, BC, CH) = 0.33 \cdot BC(t) - 0.02 \cdot BC^2(t) - 0.01 \cdot BC(t) \cdot CH(t) + 0.05 \cdot C(t) = 0$$

$$f_3(C, BC, CH) = 0.85 \cdot CH(t) - 0.02 \cdot CH^2(t) - 0.01 \cdot BC(t) \cdot CH(t) + 0.08 \cdot C(t) = 0$$

Using Maple to solve this nonlinear system numerically, we find that the fixed points are (0, 0, 0), (0, 0, 42.5), (0, 16.5, 0), and (2.772, 1.199, 42.164).

The stability of the fixed points is determined by the sign of the eigenvalues of the Jacobian [16]:

$$J(C, BC, CH) = \begin{bmatrix} \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial BC} & \frac{\partial f_1}{\partial CH} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial BC} & \frac{\partial f_2}{\partial CH} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial BC} & \frac{\partial f_3}{\partial CH} \end{bmatrix}$$

If all the eigenvalues are negative, the point is stable, otherwise, the fixed point is unstable. This indicates whether the system tends toward or away from any of these points given the appropriate initial conditions and hence whether we could expect the scenarios they represent to occur. For each fixed point, to find the corresponding eigenvalues, we have to solve for the roots of a cubic polynomial. Using Maple, we found the eigenvalues given in Table 2.

(0, 0, 0) is an unstable node due to all three eigenvalues being positive.

(0, 0, 42.5) is unstable as well as it has one positive and two negative eigenvalues. (0, 16.5, 0) is similar as it has one negative eigenvalue and two with positive real parts. These points likely act as saddle nodes.

Finally, (2.772, 1.199, 42.164) must be a stable node due to the three negative eigenvalues.

Table 2. Eigenvalues of the Fixed Points of the System Given in Equations (1)-(3).

Fixed Point	Coordinates	Eigenvalues
P_1	(0, 0, 0)	0.85, 0.6, 0.33
P_2	(0, 0, 42.5)	0.592, -0.85, -0.087
P_3	(0, 16.5, 0)	0.643+0.039i, 0.643-0.039i, -0.33
P_4	(2.772, 1.199, 42.164)	-0.854, -0.497, -0.146

In the next section, we present a computer simulation in Maple to show the behaviour of the system.

Discussion

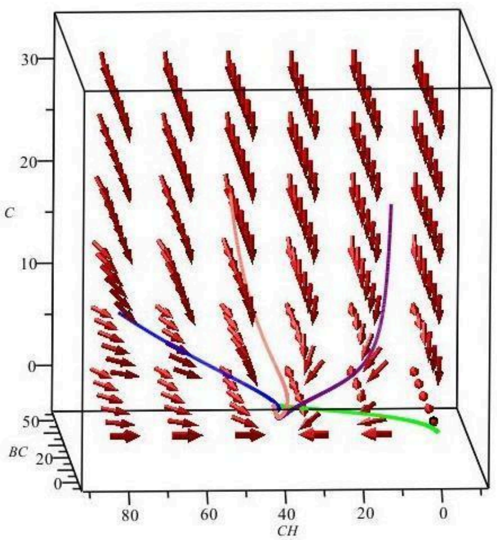

Figure 1a. Behaviour of the mathematical model given in equations (1)-(3). The purple, pink, green, and blue trajectories correspond to the initial conditions (20, 20, 10), (20, 30, 50), (0.1, 0.01, 0.1), and (10, 17, 80), respectively.

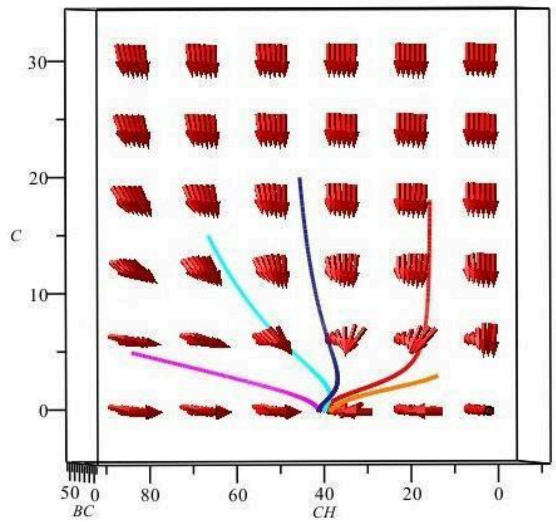

Figure 1b. Behaviour of the mathematical model given in equations (1)-(3). The orange, magenta, red, cyan, and dark blue trajectories correspond to the initial conditions (3, 25, 10), (5, 30, 80), (18, 40, 10), (15, 50, 60), and (20, 40, 40), respectively.

Figure 1a shows the behaviour of the system's fixed points using a few different initial conditions. Each trajectory clearly tends toward the stable node. However, from Figure 1b, we see that this does not occur for all initial conditions. The trajectories shown approach negative infinity in terms of the variable *C.* Figure 2 provides a better look at the behaviour of the system with respect to this variable using the initial conditions from Figure 1a and 1b.

Based on these results, the number of coyotes, number of bold coyotes, and number of concerned individuals in Edmonton could stabilize under certain initial conditions, or we could see a local extinction of the coyote population. In [7], the data collected showed that coyote boldness and human concern were on the rise, with the number of reports made involving bold coyotes and concerned individuals increasing with each year. However, the level of boldness and concern cannot increase indefinitely as the number of coyotes will reach a maximum value due to resource constraints [6]. Our results then align with these findings.

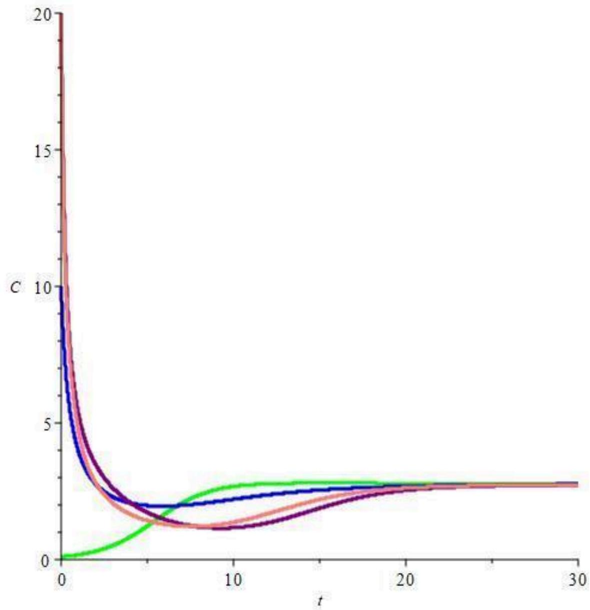

Figure 2a. Asymptotic behaviour of variable *C*. The initial conditions of each trajectory are the same as in Figure 1a.

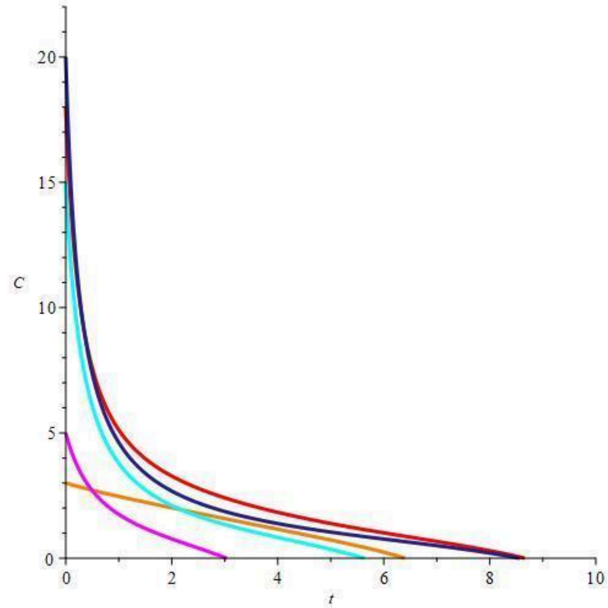

Figure 2b. Asymptotic behaviour of variable *C*. The initial conditions of each trajectory are the same as in Figure 1b.

Comparing the initial conditions from Figure 1a and 1b shows that when the initial value of *BC* is large compared to the initial value of *C*, we see a local extinction of the coyote population. This suggests that having a large amount of bold coyotes would lead to a local extinction. This makes sense as a large presence of bold coyotes would lead to many instances of lethal removal, decreasing the population. If enough coyotes are removed, the population will eventually decrease to 0 [5].

Summary and Conclusion

In this study, we created a mathematical model to study the issue of human-coyote conflict in Edmonton. While data on levels of coyote boldness and human concern from 2012-2021 had been collected and analyzed [7], we used our model to predict what these levels may be in the future.

Using computer simulations, we found that the total number of coyotes, the number of bold coyotes, and the number of concerned individuals will stabilize under certain conditions. This finding aligned with the results seen in [7], representing an end point to the growth of coyote boldness and human concern that this study found. However, if the proportion of bold coyotes is too high, the resulting amount of lethal removals will drive the local coyote population to extinction.

According to these results, we are unable to fully eliminate bold coyote behaviour or human concern without a local extinction of the coyote population. This shows that our current methods for reducing boldness are not sufficient. In order to resolve this issue, the city may need to place a greater emphasis on hazing coyotes. This approach has already started being used in Edmonton, with citizens volunteering to participate in a hazing study that took place in 2021 and 2022 [10]. This yielded promising results [14], so it may be beneficial to continue and expand upon these efforts. In particular, it is important to educate as many individuals as possible on hazing and encourage its implementation [2]. Citizens are often uncomfortable with the idea of hazing [2], so a focus should be placed on identifying ways to make it seem more appealing.

Our study is limited in that we only examined the behaviour of our system for one set of coefficients. Changing the value of some of the parameters would allow us to gain a more complete picture of the system's behaviour and may reveal other possible end behaviours than what was identified in this study. In particular, changing the parameters to reflect changes in management tactics could help show the possible outcome of these changes. This is important information which could inform decision-making about how best to handle the issue of bold coyotes in the city.

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